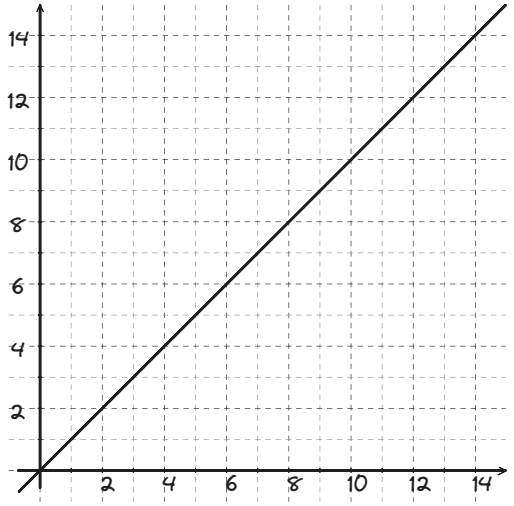
Group activity. Work on and turn in the following problems in groups of 3 or 4.

Problem 1. (Single-population) "Does a strong cup of tea really increase a person's pulse rate?"

Two people are sat down for ten minutes, then have their pulse rates (number of beats per minute) measured. They are then given a strong cup of tea, and their pulse rates are measured. The change in pulse rates are recorded as $(y_1, y_2)^T = (13, 15)^T$.

(a) Write down H_0 and H_a for this study.
(b) Write down the fitted linear model.
(c) Write down the ANOVA table by hand.
(d) Draw the decomposition of $(y_1, y_2)^T = (13, 15)^T$ into orthogonal components along and perpendicular to the vector $(1, 1)^T$:



(e) Show how the p-value is computed geometrically by hand.

Problem 2. (Two-populations)

```
data points;
    infile datalines dsd;
    input treatment $ response isControl isTreatment;
datalines;
c, 12, 1, 0
c, 13, 1, 0
c, 10, 1, 0
t, 13, 0, 1
t, 16, 0, 1
t, 11, 0, 1;
```

Fill in the blank for what you get if the following pieces of code are run:

```
proc reg data=points;
    model response = isControl;
run;
```

Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept			1.20185	11.09	0.0004
isControl			1.69967	-0.98	0.3823

```
proc reg data=points;
    model response = isTreatment;
run;
```

Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept			1.20185	9.71	0.0006
isTreatment			1.69967	0.98	0.3823

Problem 3. (Two-populations)

To determine the effect of phosphate fertilizer application on the yield of a mixed ryegrass/white clover pasture, a uniform area of pasture was chosen, and six $8\text{m} \times 3\text{m}$ experimental plots were marked out. These were assigned in a completely random manner to the two treatments:

1. No fertilizer application (control)
2. Phosphate (P) at the rate of $40\ \text{kg P/ha}$

During the winter the phosphate was spread uniformly over the appropriate plots. Throughout the following growing season the central $6\text{m} \times 2\text{m}$ of each plot was cut at monthly intervals, with clippings being weighed and returned to each plot, except for a small sample which was oven dried to determine the percentage of dry matter.

The dry matter yields for each plot, converted to tonnes/ha, were as follows:

Treatment	Plot 1	Plot 2	Plot 3
1. Control	9.6	7.4	10.0
2. Phosphate	11.3	10.1	12.2

(a) Put these six values into an observation vector y, and calculate the orthogonal decomposition $y - \bar{y}_{\cdot\cdot} = (\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot}) + (y - \bar{y}_{i\cdot})$.
(b) Calculate the Pythagorean breakup

$$\|y - \bar{y}_{\cdot\cdot}\|^2 = \|\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot}\|^2 + \|y - \bar{y}_{i\cdot}\|^2$$

and the test statistic, F. Summarize your results in an **ANOVA table**.

Source of Variation (Subspace)	Degrees of Freedom (Dimension)	Sum of Squares (Projected length squared)	Mean Squares	F
Treatment				
Error				-
Total (Corrected)			-	-

(c) Assuming the data are independent values from normal populations with means μ_1 and μ_2 and a common variance σ^2, test the hypothesis $H_0 : \mu_1 = \mu_2$ by comparing the calculated F value. What is your conclusion? Some F-statistics you can refer to:

df_1	df_2	$F_{\mathrm{df}_1,\mathrm{df}_2}$	p
1	3	4.73	0.118
1	4	4.73	0.095
1	5	4.73	0.082
1	6	4.73	0.073

(d) Draw a histogram of the errors, using the error vector, $y - \bar{y}_{i\cdot}$, and visually check for obvious problems.